EXAMPLE 17

(a) $2^8 \times 8^2 = 2^8 \times (2^3)^2 = 2^8 \times 2^6 = 2^{14}$

(b) $$\frac{x^{-2} - y^{-2}}{x^{-1} + y^{-1}} = \frac{\dfrac{1}{x^2} - \dfrac{1}{y^2}}{\dfrac{1}{x} + \dfrac{1}{y}} = \frac{\dfrac{y^2 - x^2}{x^2y^2}}{\dfrac{y + x}{xy}} = \frac{y^2 - x^2}{x^2y^2} \cdot \frac{xy}{y + x}$$

$$= \frac{(y - x)(y + x)}{xy(y + x)} = \frac{y - x}{xy}$$

(c) $4^{3/2} = \sqrt{4^3} = \sqrt{64} = 8$ Alternative solution: $4^{3/2} = (\sqrt{4})^3 = 2^3 = 8$

(d) $\dfrac{1}{\sqrt[3]{x^4}} = \dfrac{1}{x^{4/3}} = x^{-4/3}$

(e) $\left(\dfrac{x}{y}\right)^3 \left(\dfrac{y^2x}{z}\right)^4 = \dfrac{x^3}{y^3} \cdot \dfrac{y^8x^4}{z^4} = x^7y^5z^{-4}$ ■

Exercises

A Click here for answers.

1–16 ■ Expand and simplify.

1. $(-6ab)(0.5ac)$
2. $-(2x^2y)(-xy^4)$
3. $2x(x - 5)$
4. $(4 - 3x)x$
5. $-2(4 - 3a)$
6. $8 - (4 + x)$
7. $4(x^2 - x + 2) - 5(x^2 - 2x + 1)$
8. $5(3t - 4) - (t^2 + 2) - 2t(t - 3)$
9. $(4x - 1)(3x + 7)$
10. $x(x - 1)(x + 2)$
11. $(2x - 1)^2$
12. $(2 + 3x)^2$
13. $y^4(6 - y)(5 + y)$
14. $(t - 5)^2 - 2(t + 3)(8t - 1)$
15. $(1 + 2x)(x^2 - 3x + 1)$
16. $(1 + x - x^2)^2$

17–28 ■ Perform the indicated operations and simplify.

17. $\dfrac{2 + 8x}{2}$
18. $\dfrac{9b - 6}{3b}$
19. $\dfrac{1}{x + 5} + \dfrac{2}{x - 3}$
20. $\dfrac{1}{x + 1} + \dfrac{1}{x - 1}$
21. $u + 1 + \dfrac{u}{u + 1}$
22. $\dfrac{2}{a^2} - \dfrac{3}{ab} + \dfrac{4}{b^2}$
23. $\dfrac{x/y}{z}$
24. $\dfrac{x}{y/z}$
25. $\left(\dfrac{-2r}{s}\right)\left(\dfrac{s^2}{-6t}\right)$
26. $\dfrac{a}{bc} \div \dfrac{b}{ac}$
27. $\dfrac{1 + \dfrac{1}{c - 1}}{1 - \dfrac{1}{c - 1}}$
28. $1 + \dfrac{1}{1 + \dfrac{1}{1 + x}}$

29–48 ■ Factor the expression.

29. $2x + 12x^3$
30. $5ab - 8abc$
31. $x^2 + 7x + 6$
32. $x^2 - x - 6$
33. $x^2 - 2x - 8$
34. $2x^2 + 7x - 4$
35. $9x^2 - 36$
36. $8x^2 + 10x + 3$
37. $6x^2 - 5x - 6$
38. $x^2 + 10x + 25$
39. $t^3 + 1$
40. $4t^2 - 9s^2$
41. $4t^2 - 12t + 9$
42. $x^3 - 27$
43. $x^3 + 2x^2 + x$
44. $x^3 - 4x^2 + 5x - 2$
45. $x^3 + 3x^2 - x - 3$
46. $x^3 - 2x^2 - 23x + 60$
47. $x^3 + 5x^2 - 2x - 24$
48. $x^3 - 3x^2 - 4x + 12$

49–54 ■ Simplify the expression.

49. $\dfrac{x^2 + x - 2}{x^2 - 3x + 2}$
50. $\dfrac{2x^2 - 3x - 2}{x^2 - 4}$
51. $\dfrac{x^2 - 1}{x^2 - 9x + 8}$
52. $\dfrac{x^3 + 5x^2 + 6x}{x^2 - x - 12}$
53. $\dfrac{1}{x + 3} + \dfrac{1}{x^2 - 9}$

54. $\dfrac{x}{x^2 + x - 2} - \dfrac{2}{x^2 - 5x + 4}$

55–60 ■ Complete the square.

55. $x^2 + 2x + 5$

56. $x^2 - 16x + 80$

57. $x^2 - 5x + 10$

58. $x^2 + 3x + 1$

59. $4x^2 + 4x - 2$

60. $3x^2 - 24x + 50$

61–68 ■ Solve the equation.

61. $x^2 + 9x - 10 = 0$

62. $x^2 - 2x - 8 = 0$

63. $x^2 + 9x - 1 = 0$

64. $x^2 - 2x - 7 = 0$

65. $3x^2 + 5x + 1 = 0$

66. $2x^2 + 7x + 2 = 0$

67. $x^3 - 2x + 1 = 0$

68. $x^3 + 3x^2 + x - 1 = 0$

69–72 ■ Which of the quadratics are irreducible?

69. $2x^2 + 3x + 4$

70. $2x^2 + 9x + 4$

71. $3x^2 + x - 6$

72. $x^2 + 3x + 6$

73–76 ■ Use the Binomial Theorem to expand the expression.

73. $(a + b)^6$

74. $(a + b)^7$

75. $(x^2 - 1)^4$

76. $(3 + x^2)^5$

77–82 ■ Simplify the radicals.

77. $\sqrt{32}\sqrt{2}$

78. $\dfrac{\sqrt[3]{-2}}{\sqrt[3]{54}}$

79. $\dfrac{\sqrt[4]{32x^4}}{\sqrt[4]{2}}$

80. $\sqrt{xy}\sqrt{x^3y}$

81. $\sqrt{16a^4b^3}$

82. $\dfrac{\sqrt[5]{96a^6}}{\sqrt[5]{3a}}$

83–100 ■ Use the Laws of Exponents to rewrite and simplify the expression.

83. $3^{10} \times 9^8$

84. $2^{16} \times 4^{10} \times 16^6$

85. $\dfrac{x^9(2x)^4}{x^3}$

86. $\dfrac{a^n \times a^{2n+1}}{a^{n-2}}$

87. $\dfrac{a^{-3}b^4}{a^{-5}b^5}$

88. $\dfrac{x^{-1} + y^{-1}}{(x + y)^{-1}}$

89. $3^{-1/2}$

90. $96^{1/5}$

91. $125^{2/3}$

92. $64^{-4/3}$

93. $(2x^2y^4)^{3/2}$

94. $(x^{-5}y^3z^{10})^{-3/5}$

95. $\sqrt[5]{y^6}$

96. $(\sqrt[4]{a})^3$

97. $\dfrac{1}{(\sqrt{t})^5}$

98. $\dfrac{\sqrt[8]{x^5}}{\sqrt[4]{x^3}}$

99. $\sqrt[4]{\dfrac{t^{1/2}\sqrt{st}}{s^{2/3}}}$

100. $\sqrt[4]{r^{2n+1}} \times \sqrt[4]{r^{-1}}$

101–108 ■ Rationalize the expression.

101. $\dfrac{\sqrt{x} - 3}{x - 9}$

102. $\dfrac{(1/\sqrt{x}) - 1}{x - 1}$

103. $\dfrac{x\sqrt{x} - 8}{x - 4}$

104. $\dfrac{\sqrt{2 + h} + \sqrt{2 - h}}{h}$

105. $\dfrac{2}{3 - \sqrt{5}}$

106. $\dfrac{1}{\sqrt{x} - \sqrt{y}}$

107. $\sqrt{x^2 + 3x + 4} - x$

108. $\sqrt{x^2 + x} - \sqrt{x^2 - x}$

109–116 ■ State whether or not the equation is true for all values of the variable.

109. $\sqrt{x^2} = x$

110. $\sqrt{x^2 + 4} = |x| + 2$

111. $\dfrac{16 + a}{16} = 1 + \dfrac{a}{16}$

112. $\dfrac{1}{x^{-1} + y^{-1}} = x + y$

113. $\dfrac{x}{x + y} = \dfrac{1}{1 + y}$

114. $\dfrac{2}{4 + x} = \dfrac{1}{2} + \dfrac{2}{x}$

115. $(x^3)^4 = x^7$

116. $6 - 4(x + a) = 6 - 4x - 4a$

Answers

1. $-3a^2bc$ 2. $2x^3y^5$ 3. $2x^2 - 10x$ 4. $4x - 3x^2$
5. $-8 + 6a$ 6. $4 - x$ 7. $-x^2 + 6x + 3$
8. $-3t^2 + 21t - 22$ 9. $12x^2 + 25x - 7$
10. $x^3 + x^2 - 2x$ 11. $4x^2 - 4x + 1$
12. $9x^2 + 12x + 4$ 13. $30y^4 + y^5 - y^6$
14. $-15t^2 - 56t + 31$ 15. $2x^3 - 5x^2 - x + 1$
16. $x^4 - 2x^3 - x^2 + 2x + 1$ 17. $1 + 4x$ 18. $3 - 2/b$
19. $\dfrac{3x + 7}{x^2 + 2x - 15}$ 20. $\dfrac{2x}{x^2 - 1}$ 21. $\dfrac{u^2 + 3u + 1}{u + 1}$
22. $\dfrac{2b^2 - 3ab + 4a^2}{a^2b^2}$ 23. $\dfrac{x}{yz}$ 24. $\dfrac{zx}{y}$ 25. $\dfrac{rs}{3t}$
26. $\dfrac{a^2}{b^2}$ 27. $\dfrac{c}{c - 2}$ 28. $\dfrac{3 + 2x}{2 + x}$ 29. $2x(1 + 6x^2)$
30. $ab(5 - 8c)$ 31. $(x + 6)(x + 1)$ 32. $(x - 3)(x + 2)$
33. $(x - 4)(x + 2)$ 34. $(2x - 1)(x + 4)$
35. $9(x - 2)(x + 2)$ 36. $(4x + 3)(2x + 1)$
37. $(3x + 2)(2x - 3)$ 38. $(x + 5)^2$
39. $(t + 1)(t^2 - t + 1)$ 40. $(2t - 3s)(2t + 3s)$
41. $(2t - 3)^2$ 42. $(x - 3)(x^2 + 3x + 9)$
43. $x(x + 1)^2$ 44. $(x - 1)^2(x - 2)$
45. $(x - 1)(x + 1)(x + 3)$ 46. $(x - 3)(x + 5)(x - 4)$
47. $(x - 2)(x + 3)(x + 4)$ 48. $(x - 2)(x - 3)(x + 2)$
49. $\dfrac{x + 2}{x - 2}$ 50. $\dfrac{2x + 1}{x + 2}$ 51. $\dfrac{x + 1}{x - 8}$ 52. $\dfrac{x(x + 2)}{x - 4}$
53. $\dfrac{x - 2}{x^2 - 9}$ 54. $\dfrac{x^2 - 6x - 4}{(x - 1)(x + 2)(x - 4)}$
55. $(x + 1)^2 + 4$ 56. $(x - 8)^2 + 16$ 57. $\left(x - \frac{5}{2}\right)^2 + \frac{15}{4}$
58. $\left(x + \frac{3}{2}\right)^2 - \frac{5}{4}$ 59. $(2x + 1)^2 - 3$
60. $3(x - 4)^2 + 2$ 61. $1, -10$ 62. $-2, 4$
63. $\dfrac{-9 \pm \sqrt{85}}{2}$ 64. $1 \pm 2\sqrt{2}$ 65. $\dfrac{-5 \pm \sqrt{13}}{6}$
66. $\dfrac{-7 \pm \sqrt{33}}{4}$ 67. $1, \dfrac{-1 \pm \sqrt{5}}{2}$ 68. $-1, -1 \pm \sqrt{2}$
69. Irreducible 70. Not irreducible
71. Not irreducible (two real roots) 72. Irreducible
73. $a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6$
74. $a^7 + 7a^6b + 21a^5b^2 + 35a^4b^3 + 35a^3b^4 + 21a^2b^5 + 7ab^6 + b^7$
75. $x^8 - 4x^6 + 6x^4 - 4x^2 + 1$
76. $243 + 405x^2 + 270x^4 + 90x^6 + 15x^8 + x^{10}$
77. 8 78. $-\frac{1}{3}$ 79. $2|x|$ 80. $x^2|y|$
81. $4a^2b\sqrt{b}$ 82. $2a$ 83. 3^{26} 84. 2^{60} 85. $16x^{10}$
86. a^{2n+3} 87. $\dfrac{a^2}{b}$ 88. $\dfrac{(x + y)^2}{xy}$ 89. $\dfrac{1}{\sqrt{3}}$
90. $2\sqrt[5]{3}$ 91. 25 92. $\frac{1}{256}$ 93. $2\sqrt{2}\,|x|^3y^6$
94. $\dfrac{x^3}{y^{9/5}z^6}$ 95. $y^{6/5}$ 96. $a^{3/4}$ 97. $t^{-5/2}$ 98. $\dfrac{1}{x^{1/8}}$
99. $\dfrac{t^{1/4}}{s^{1/24}}$ 100. $r^{n/2}$ 101. $\dfrac{1}{\sqrt{x} + 3}$ 102. $\dfrac{-1}{\sqrt{x} + x}$
103. $\dfrac{x^2 + 4x + 16}{x\sqrt{x} + 8}$ 104. $\dfrac{2}{\sqrt{2 + h} - \sqrt{2 - h}}$
105. $\dfrac{3 + \sqrt{5}}{2}$ 106. $\dfrac{\sqrt{x} + \sqrt{y}}{x - y}$
107. $\dfrac{3x + 4}{\sqrt{x^2 + 3x + 4} + x}$ 108. $\dfrac{2x}{\sqrt{x^2 + x} + \sqrt{x^2 - x}}$
109. False 110. False 111. True 112. False
113. False 114. False 115. False 116. True